

### Purpose

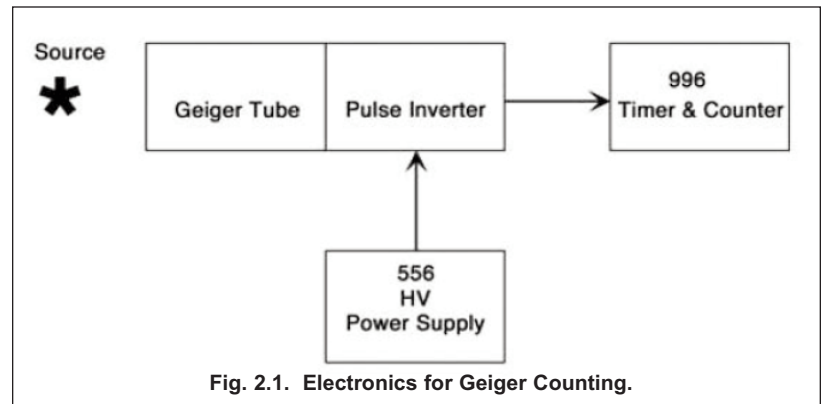
The purpose of this experiment is to familiarize the student with the Geiger-Mueller counter.<sup>1</sup> This counter is a widely used pulse-counting instrument for X-ray, gamma-ray, beta-particle and alpha-particle detection. It uses gas amplification, which makes it remarkably sensitive, while the simple construction renders it relatively inexpensive. The experiments that are designed to accomplish this purpose deal with the operating plateau of the Geiger tube, resolving-time corrections, half-life determinations, and the basic nuclear counting principles.

### Description

Basically, the Geiger counter consists of two electrodes with a gas at reduced pressure between the electrodes. The outer electrode is usually a cylinder, while the inner (positive) electrode is a thin wire positioned in the center of the cylinder. The voltage between these two electrodes is maintained at such a value that virtually any ionizing particle entering the Geiger tube will cause a comprehensive electrical avalanche within the tube. This discharge is so massive that the output pulse has essentially the same amplitude, regardless of the type of radiation that caused it. Thus, the Geiger counter cannot measure the energy of the impinging radiation, and functions only as a means of counting the number of photons or charged particles that were detected. The voltage pulse from the avalanche is typically greater than a few Volts in amplitude. These pulses are large enough that they can be counted in an ORTEC 996 Timer and Counter without amplification.

However, pulse inversion is necessary (Fig. 2.1) because the Geiger counter output is a negative pulse, whereas the 996 Timer and Counter requires a positive pulse at its POSitive INput.

The Geiger tube used in this experiment is called an end-window tube because it has a thin window at one end through which the ionizing radiation enters. In this experiment the properties of the Geiger counter will be studied, and several fundamental measurements will be made. For more information on the Geiger counter see the chapter on Geiger-Mueller Counters in reference 1.



### Equipment Required

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| <ul style="list-style-type: none"> <li>• <b>556</b> High Voltage Bias Supply</li> <li>• <b>4001A/4002D</b> NIM Bin and Power Supply</li> <li>• <b>996</b> Timer and Counter</li> <li>• <b>C-36-12</b> 12-ft. (3.7-m) 75-Ω RG-59A/U Coaxial Cable with two SHV female plugs</li> <li>• Two <b>C-24-4</b> 4-ft. (1.2 m) 93-Ω RG-62A/U Coaxial Cables with BNC plugs</li> <li>• <b>C-29</b> BNC Tee Connector</li> <li>• <b>GP35</b> End-Window Geiger-Mueller Tube with Stand providing 6 counting levels</li> </ul> | <ul style="list-style-type: none"> <li>• <b>GPI</b> Pulse Inverter (to convert the negative G10-M tube signal to a positive pulse for the 996 input).</li> <li>• <b>TDS3032C</b> 300 MHz, 2-Channel Digital Oscilloscope.</li> <li>• Two <b>RAS20</b> Absorber Foil Kits, each containing at least 4 lead absorbers from 1100 to 7400 mg/cm<sup>2</sup></li> <li>• <b>Co60S*</b> Sealed Solid Disk Gamma Ray Source: ~1 μCi of <sup>60</sup>Co</li> <li>• <b>RSS2*</b> Resolving Time Set Split Source TI-204.</li> <li>• Small, flat-blade screwdriver for tuning screwdriver-adjustable controls</li> </ul> |
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\*Sources are available direct from supplier. See the ORTEC website at [www.ortec-online.com/Service-Support/Library/Experiments-Radioactive-Source-Suppliers.aspx](http://www.ortec-online.com/Service-Support/Library/Experiments-Radioactive-Source-Suppliers.aspx)

<sup>1</sup>The traditional name of this detector includes the names of the two inventors of the device. For convenience, the name is often shortened to Geiger Counter or Geiger Tube.

## Experiment 2 Geiger Counting

### EXPERIMENT 2.1. Operating Plateau for the Geiger Tube

#### Purpose

The purpose of this experiment is to determine the voltage plateau for the Geiger tube and to establish a reasonable operating point for the tube. Fig. 2.2 shows a counts-vs.-voltage curve for a typical Geiger tube that has an operating point in the vicinity of 950 V.

The region between  $R_1$  and  $R_2$ , corresponding to operating voltages  $V_1$  and  $V_2$ , is called the Geiger region. Voltages  $>V_2$  in Fig. 2.2 cause a continuous discharge in the tube and should be avoided, because a continuous discharge will definitely shorten the life of the tube.

#### Procedure

1. Set up the electronics as shown in Fig. 2.1. Ensure that all power is turned off.
2. On the 556 High Voltage Power Supply, set all three of the front-panel Voltage controls to their minimum value. On the rear panel, confirm that the POSitive POLARITY has been selected, and the CONTROL switch is set to INTernal.
3. Connect the cable with the MHV connector from the Geiger tube to the MHV connector labeled "GM Tube" on the GM Pulse Inverter box. Using the coaxial cable with SHV plugs, connect the high voltage OUTPUT on the rear panel of the 556 High Voltage Power Supply to the SHV connector labeled "HIGH VOLTAGE" on the GM Pulse Inverter box.
4. Using an RG-62A/U coaxial cable with BNC plugs, connect the OUTPUT on the GM Pulse Inverter box to a BNC Tee on the Channel 1 input of the oscilloscope. Connect the other side of the Tee to the POSitive INput on the 996 Timer and Counter using another RG-62A/U coaxial cable.
5. Set the oscilloscope to display the Channel 1 input on a vertical scale of 2 Volts per major division and a horizontal scale of 10  $\mu$ s per major division. Set the triggering mode to Auto, and select the Channel 1 input as the triggering source. Position the ground trace near the bottom of the display, and set the input coupling to DC. This setup should allow the oscilloscope to display the +4-V logic pulse from the GM Pulse Inverter.
6. Turn on power to the Bin and Power Supply and turn on power to the 556 High Voltage Power Supply.
7. On the 996 turn the THRESHold ADJustment screwdriver control counterclockwise until it clicks at its minimum setting. Next, turn the potentiometer clockwise 4.0 turns. This should set the POSitive INput threshold at +1.6 Volts. (The screwdriver adjustment operates a 25-turn potentiometer to set the POSitive INput threshold between +100 mV and +9.5 V.)
8. On the 996, set the TIME BASE to 0.01 MINutes. Select the PRESET time display and choose M = 3, N = 0 and P = 2. That chooses a counting time of 30 minutes.
9. Set the 996 DISPLAY to COUNTS.
10. Place the two halves of the split beta source from the source kit at a distance of  $\sim$ 2 cm from the window of the Geiger tube. Depending on the manufacturer, this may be either a  $^{90}\text{Sr}/^{90}\text{Y}$  or a  $^{204}\text{Tl}$  radioactive source.<sup>2</sup> If the active source is deposited behind a thin window on one side of each plastic half-disk, make sure that the active sides are facing the end window on the Geiger counter.
11. Gradually increase the (positive) high voltage on the 556 in steps no larger than 50 V until the 996 counter just begins registering counts and the 4-V logic pulse shows up on the oscilloscope input. This point is called the starting voltage in Fig. 2.2. Starting voltages are rarely  $>900$  V and can be as low as 250 V. If the counts begin advancing on the 996 Timer and Counter, but the pulse does not show up on the oscilloscope, adjust the oscilloscope triggering. If the pulse shows up on the oscilloscope, but the

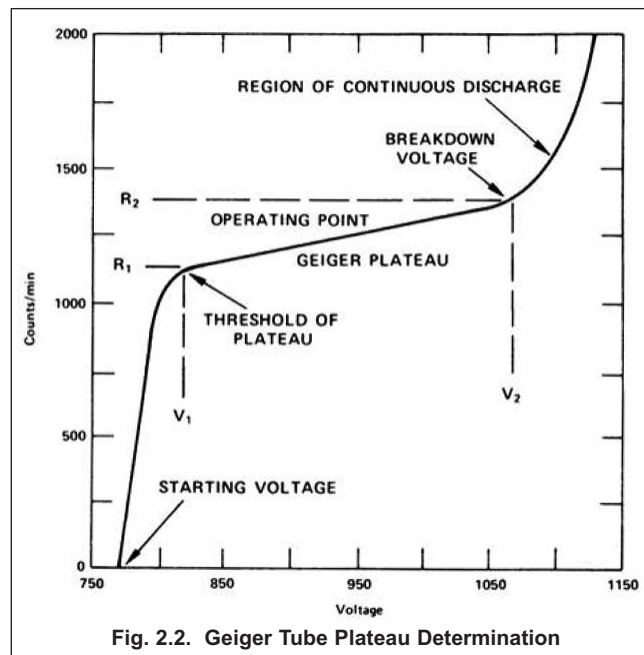


Fig. 2.2. Geiger Tube Plateau Determination

<sup>2</sup> $^{90}\text{Sr}$  has a half-life of 28 years, and decays to  $^{90}\text{Y}$  by  $\beta^-$  emission, with an endpoint energy of 0.546 MeV. The daughter nucleus,  $^{90}\text{Y}$ , has a 64-hr. half-life, and decays to the stable nucleus  $^{90}\text{Zr}$  more than 99% of the time by  $\beta^-$  emission, with an endpoint energy of 2.27 MeV.  $^{204}\text{Tl}$  has a half-life of 3.8 years, and decays to the stable nucleus  $^{204}\text{Pb}$  by  $\beta^-$  decay, with an endpoint energy of 0.766 MeV. For more details consult reference 7.

996 does not begin counting, check the setup of the 996. Basically, the oscilloscope is being used to check that the electronics are operating properly. Once proper triggering has been established, the triggering mode on the oscilloscope can be switched from Auto to Normal, with a subsequent readjustment of the triggering threshold.

12. Reset the 996 counter. Set the timer section for 1-minute time intervals, and count for 1 minute. Record the number of counts.
13. Increase the high voltage by 50 V and count again for 1 minute. Record the number of counts.

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### EXERCISES

- a. Continue making measurements at 50-V intervals until you have enough data to plot a curve on linear graph paper similar to that in Fig. 2.2 (**CAUTION:** use only values below V<sub>2</sub>). The length of the region between V<sub>1</sub> and V<sub>2</sub> is usually <300 V. A sharp rise in the counting rate will be observed if you go just above V<sub>2</sub>. When this happens, the upper end of the plateau has been reached. Reduce the voltage to V<sub>2</sub> immediately. Choose the operating point for your instrument at ~40 to 60% of the plateau range. In other words, the Geiger counter should be operated at the midpoint of the plateau.
- b. Evaluate your Geiger tube by measuring the slope of the plateau in the graph; a good result is 30% per 100 V. The slope of the plateau is defined as:

$$\text{Slope (in \% per 100 V)} = \left[ \frac{R_2 - R_1}{R_1} \right] \left[ \frac{100}{V_2 - V_1} \right] \times 100\% \quad (1)$$

- c. Record the operating voltage selected for use in the remainder of the experiments.
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## EXPERIMENT 2.2. Resolving-Time Corrections for the Geiger Counter

### Purpose

Later experiments will be dealing with fast electronics capable of resolving sequential events spaced as closely as a few tens of nanoseconds. In stark contrast, the Geiger counter is very slow in responding to detected events. It takes of the order of a microsecond for the detector to develop its full response to the incident gamma-ray or charged particle, and it requires hundreds of microseconds to restore the detector to full sensitivity for the next event. The strict definition of the *dead time* of the Geiger counter is the time from initial response to a detected event until the detector can exhibit the earliest, albeit crippled, response to a subsequent event. But, the electronics processing the Geiger tube output pulses has a fixed voltage threshold that the pulses must exceed to be counted. Thus the *resolving time* includes the previously defined dead time, but adds the time it takes for subsequent pulses to recover to a sufficient amplitude to cross the discriminator threshold and be counted. In practice, the distinction between *dead time* and *resolving time* is often blurred, and the resolving time is frequently labeled as the dead time. That is a pragmatically empirical convention, because the measurement of counting rate is always made after the supporting electronics has added its contribution to the dead time.

The large dead time of the Geiger counter distorts the measured counting rate for counting rates above 5000 counts/minute. Thus, it is usually necessary to make a dead-time correction to obtain the true counting rate. In this experiment the measurement of the dead time will be accomplished with a split source. The measured dead time will be employed to correct the counting rates in all the subsequent measurements.

### Relevant Equations

The dead time of a nuclear radiation counting system is typically dominated by one of two types of dead time: 1) paralyzable (a.k.a., extending) dead time, or 2) non-paralyzable (a.k.a., non-extending) dead time. For a full explanation see references 1 and 11. The dead time contributed by the Geiger counter is reasonably accurately modeled as a non-paralyzable dead time. The measured counting rate, R, is related to the true counting rate, r, at the input to the detector via equation (2).

$$R = \frac{r}{1 + rT_d} \quad (2)$$

Where T<sub>d</sub> is the dead time caused by each quantum of radiation that is detected when the counter is free to accept a new event. Note that the dead time reduces the measured counting rate relative to the true counting rate, and higher counting rates cause a greater relative reduction.

## Experiment 2

### Geiger Counting

In practice, the experimenter only has access to the measured counting rate,  $R$ , after dead time losses have occurred. Consequently, it is important to calculate the true counting rate, and this requires knowing the dead time per pulse,  $T_d$ . If equation (2) is rearranged to the form in equation (3), one can compute the true counting rate from the measured counting rate and the known dead time per pulse.

$$r = \frac{R}{1 - RT_d} \quad (3)$$

Another useful way to express the information in equations (2) and (3) is the percent dead time loss:

$$\text{Percent Dead Time} = \frac{r - R}{r} \times 100\% = RT_d \times 100\% = \frac{rT_d}{1 + rT_d} \times 100\% \quad (4)$$

One way to measure the dead time per pulse,  $T_d$ , would be to observe the output of the GM Pulse Inverter on the oscilloscope and determine the minimum spacing between the leading edges of two successive pulses. Because the arrival times of the pulses are randomly distributed in time, this method requires a fairly high counting rate to make it easy to find the minimum pulse spacing. There is a risk that the excessively high counting rate changes the dead time per pulse compared to what would be experienced at the counting rates normally used to assay the activity of radioactive samples.

The split source method has the advantage of assessing the dead time per pulse at the counting rates normally employed in assays. In this scheme, the radioactive source is contained in a circular disc that has been sectioned into two halves. Each half contains approximately the same source activity. With both halves positioned side-by-side to form the complete circular disk, the distance of the source from the window of the Geiger tube is adjusted to achieve a percent dead time in the range of 10% to 20%. For a 100- $\mu$ s dead time, this implies measured counting rates in the range of 1,000 to 2,000 counts/s, or 60,000 to 120,000 counts per minute.

Next, the first half of the source is removed, and the counting rate of the second half,  $R_2$ , is measured. Subsequently, the first half is carefully reinstalled without disturbing the second half, and the counting rate from the pair of sources,  $R_{12}$ , is measured. Finally, the second half of the disk is removed without disturbing the first half, and the counting rate,  $R_1$ , is measured.  $R_1$ ,  $R_2$ , and  $R_{12}$  can be inserted into equation (3) to write the equations for  $r_1$ ,  $r_2$ , and  $r_{12}$ , respectively. Because of the dead time,  $R_{12} < (R_1 + R_2)$ . But for the true counting rates

$$r_{12} = r_1 + r_2 \quad (5)$$

Combining equation (5) with the expressions for  $r_1$ ,  $r_2$  and  $r_{12}$  from equation (3), permits solving for  $T_d$  in terms of the measured counting rates. From reference 1, the exact solution for the case of zero background is

$$T_d = \frac{R_1 R_2 - [R_1 R_2 (R_{12} - R_1)(R_{12} - R_2)]^{1/2}}{R_1 R_2 R_{12}} \quad (6)$$

An approximate solution that is sometimes employed is

$$T_d \approx \frac{R_1 + R_2 - R_{12}}{2R_1 R_2} \quad (7)$$

### Procedure

1. Place both halves of the radioactive source split disk on the sample-holder shelf, with the disk centered below the Geiger counter window. Set the 996 Timer and Counter for a 1-minute counting interval.
2. Measure the counts for 1 minute.
3. If the number of counts in step 2 is not between 60,000 and 120,000, adjust the source to detector distance to bring the counting rate within that range by repeating steps 1 and 2.
4. Remove the left half of the split source and make a 1-minute count on the right half. Record the count. Define this count to be  $R_1$ .
5. Being careful not to disturb the right half, place the left half of the source alongside the right half and make a 1 minute count. Define this count to be  $R_{12}$ .
6. Being careful not to disturb the left half, remove the right half and count the left half for 1 minute. Define this count to be  $R_2$ . Calculate the resolving time of the Geiger tube via equation (6). The answer should be in minutes/count. Because counts are often considered a dimensionless number, the dead time can also be expressed simply in minutes or seconds.

The dead time established in step 6 should be used to correct all measured counting rates via equation (3) whenever the percent dead time exceeds 1%.

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**EXERCISES**

- Calculate the value of  $T_d$  from equation (7) and compare it to the result from equation (6). What is the percent error in the approximate result from equation (7) compared to the more exact result from equation (6)?
  - Based on equation (4), what is the measured counting rate that will correspond to a 1% dead time? Correct all subsequent measurements above that counting rate for the dead time loss.
  - Considering the apparatus and procedures employed in this measurement, what are the most important sources of error in measuring  $T_d$ ?
  - Using the oscilloscope, determine the minimum observable time between the leading edge of successive pulses. It may be useful to use both half-disks and move the sources closer to the Geiger counter window for this measurement. What is the percent difference between this minimum separation and the value of  $T_d$  from equation (6)?
  - Some split source kits include a blank half-disk to replace the removed source half. The idea is that each half-disk scatters some of the radiation from the other half-disk into the Geiger counter. The blank half-disk duplicates that scattering when either of the active halves is removed. If your source kit includes the blank half disk, an optional measurement is to employ the blank, and determine whether it changes the measured dead time.
  - BONUS QUESTION:** Remove both active halves of the split source and measure the background counts for one minute. Based on the equations in reference 1, would correcting for the background make a difference in the computed value for  $T_d$ ?
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## EXPERIMENT 2.3. Half-Life Determination

### Purpose

The purpose of this experiment is to construct a decay curve and determine the half-life of an unknown isotope. The instructor will provide the unknown short half-life source to be used for this experiment. You will be advised at what time intervals counts are to be made, and the recommended duration for the counting time. For example, the instructions might be to take one 10-minute measurement every hour for the next 6 hours, or one 2-minute measurement every 15 minutes for the next 3 hours.

### Relevant Equations

Radioisotopes decay randomly in time. It is not possible to predict when a specific nucleus will decay. However, when a very large ensemble of  $N$  atoms of a specific radioisotope are present, it is possible to predict the probability that one of the nuclei will decay in an infinitesimal increment of time. The probability of decay per unit time,  $dN/dt$ , is proportional to the number of radioactive nuclei present.

$$\frac{dN}{dt} = -\lambda N \quad (8)$$

Where  $\lambda$  is the decay constant, and is characteristic of the particular radioisotope. The number of decays per second,  $dN/dt$ , is also known as the activity,  $A$ .

Solving the differential equation (8) leads to the equation describing the exponential decrease in activity as a function of time.

$$A = A_0 e^{-\lambda t} \quad (9)$$

Where  $A_0$  is the activity at time  $t = 0$ . The activity can be measured in units of disintegrations per second (dps), Becquerels, or Curies (Ci). One Becquerel is one disintegration per second, and 1 Curie is equivalent to  $3.7 \times 10^{10}$  disintegrations per second.

When the half-life,  $T_{1/2}$ , of the radioisotope is defined as the time it takes for the activity to decrease to 1/2 of its former value,  $\lambda$  can be replaced by

$$\lambda = \frac{0.693}{T_{1/2}} \quad (10)$$

## Experiment 2 Geiger Counting

The Geiger counter employed in this experiment intercepts only a fraction of the radiation emitted by the radioactive source, because of the small solid angle subtended, and the efficiency of the detector. However, the true counting rate,  $r$ , is still proportional to the activity.

$$r = r_0 \exp(-0.693 t/T_{1/2}) \quad (11)$$

Where  $r_0$  is the counting rate at time  $t = 0$ .

Taking the natural logarithm of both sides of equation (11) leads to a linear equation:

$$\ln r = \ln r_0 - \frac{0.693}{T_{1/2}} t \quad (12)$$

Thus, plotting the true counting rate versus time on semilog graph paper will yield a straight line. The decay constant and the half-life can be derived from the slope of that line.

### Procedure

1. Set the Geiger tube at the operating voltage determined in Experiment 2.1.
2. Place the unknown half-life source 2 cm away from the Geiger tube window and make a count as in Experiment 2.1.
3. Record the time of day, counting duration, and number of counts.
4. After the period of time recommended by the laboratory instructor, repeat the measurement. Be sure to place the sample at exactly the same distance from the Geiger tube window.
5. Continue the measurements at the time intervals recommended by the instructor. When you are not making half-life measurements, you can continue with the other sub-sections of Experiment 2.

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### EXERCISES

- a. When you have completed your half-life measurements, correct the counting rates for dead-time losses (see Experiment 2.2), and plot the corrected counting rates as a function of time on semilog graph paper. A straight line should result.
  - b. Determine the half-life from the curve and find  $\lambda$ , the decay constant for the isotope.
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## EXPERIMENT 2.4. Linear Absorption Coefficient

### Purpose

When gamma radiation passes through matter, it undergoes absorption primarily by Compton, photoelectric, and pair-production interactions. The intensity of the radiation is thus decreased as a function of distance in the absorbing medium. The purpose of this experiment is to measure the attenuation of the intensity with absorber thickness, and to derive the half-thickness and the attenuation coefficient.

### Relevant Equations

The mathematical expression for the surviving intensity,  $I$ , is given by the following:

$$I = I_0 e^{-\mu x} \quad (13)$$

Where

$I_0$  = original intensity of the beam,

$I$  = intensity transmitted through an absorber to a distance, depth, or thickness,  $x$ ,

$\mu$  = linear absorption coefficient for the absorbing medium.



If we rearrange Eq. (13) and take the logarithm of both sides, the expression becomes

$$\ln\left(\frac{I}{I_0}\right) = -\mu x \quad (14)$$

The half-value layer (HVL) of the absorbing medium is defined as that thickness,  $x_{1/2}$ , which will cut the initial intensity in half. That is,  $I/I_0 = 0.5$ . If we substitute this into Eq. (14),

$$\ln(0.5) = -\mu x_{1/2} \quad (15)$$

Putting in numerical values and rearranging, Eq. (15) becomes

$$x_{1/2} = \frac{0.693}{\mu} \quad \text{or} \quad \mu = \frac{0.693}{x_{1/2}} \quad (16)$$

Experimentally, the usual procedure is to measure  $x_{1/2}$  and then calculate  $\mu$  from Eq. (16). If the thickness of the absorber is expressed in cm, then the units of  $\mu$  are  $\text{cm}^{-1}$ , and it is known as the linear attenuation coefficient. Often, the thickness of the absorber is expressed in  $\text{g}/\text{cm}^2$ . In that case, the attenuation coefficient has units of  $\text{cm}^2/\text{g}$ , and is identified as the mass attenuation coefficient.

### Procedure

1. Set the voltage of the Geiger tube at the operating value determined in Experiment 2.1.
2. Place the  $^{60}\text{Co}$  source about 3 cm from the window of the Geiger tube, and make a 2-minute count. Record the number of counts.
3. Note the various thicknesses of the lead sheets in the absorber kit. They may all be of equal thickness, or they may have nominal thicknesses of 1,000  $\text{mg}/\text{cm}^2$ , 2,000  $\text{mg}/\text{cm}^2$ , 3,000  $\text{mg}/\text{cm}^2$  and 7,000  $\text{mg}/\text{cm}^2$ . This experiment will require a total thickness ranging from 1,000  $\text{mg}/\text{cm}^2$  to circa 23,000  $\text{mg}/\text{cm}^2$  in 1,000 to 2,000  $\text{mg}/\text{cm}^2$  steps. Incrementing the total thickness will require using the various foils in suitable combinations to achieve the desired thickness increments.
4. Place the thinnest sheet of lead from the absorber kit between the source and the GM tube window and take another 2-minute count. Record the value.
5. Add a second sheet of lead on top of the first to increase the total thickness by a value between 1,000 and 2,000  $\text{mg}/\text{cm}^2$ , and make another count.
6. Continue inserting combinations of lead sheets to increment the total thickness in steps of 1,000 to 2,000  $\text{mg}/\text{cm}^2$  until the number of counts is 25% of the number recorded with no absorber. Record the counts taken in 2 minutes for each step in the total absorber thickness.
7. Correct the measured counts for dead time whenever the dead time losses are calculated to be  $>1\%$ .
8. Make a 2-minute background run with the  $^{60}\text{Co}$  source removed to a long distance from the counting station, and subtract this value from each of the above counts that have been corrected for dead time. Check this background count at the maximum absorber thickness employed and without any absorbers. The result should be the same, or close enough to the same that the average of the two background readings can be used for background subtraction from all the corrected counting rates with the source in the counting position.

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### EXERCISE

- a. Correct all the measured counting rates for the dead time measured in Experiment 2.2, if the correction alters the result by more than 1%. This correction should be applied before the background is subtracted.
  - b. Record the total density-thickness of the lead in  $\text{g}/\text{cm}^2$  and plot on semilog paper the corrected counts as a function of absorber density-thickness in  $\text{g}/\text{cm}^2$ . The density-thickness is defined as the product of density in  $\text{g}/\text{cm}^3$  times the thickness of the absorber in cm.
  - c. Draw the best straight line through the points, and determine  $x_{1/2}$  and  $\mu$  from the slope of the line.
  - d. How do your values compare with those indicated in references 8 and 9? See also Experiment 3 in this manual, in which this same experiment is done with a sodium iodide detector.
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## Experiment 2 Geiger Counting

### EXPERIMENT 2.5. Inverse Square Law

#### Purpose and Relevant Equations

There are many similarities between ordinary light rays and gamma rays. They are both considered to be electromagnetic radiation; hence they obey the classical equation

$$E = hv \quad (17)$$

where

$E$  = energy of the photon in ergs,

$v$  = the frequency of the radiation in cycles/second,

$h$  = Planck's constant ( $6.624 \times 10^{-27}$  ergs  $\cdot$  s).

Therefore, in explaining the inverse square law it is convenient to make the analogy between a light source and a gamma-ray source.

Let us assume that we have a light source that emits light photons at a rate,  $n_0$  photons/second. It is reasonable to assume that these photons are given off in an isotropic manner, that is, equally in all directions. If we place the light source in the center of a clear plastic spherical shell, it is quite easy to measure the number of light photons per second for each  $\text{cm}^2$  of the spherical shell. This intensity is given by

$$I_0 = \frac{n_0}{4\pi r_s^2} \quad (18)$$

Where

$n_0$  is the total number of photons per second from the source,

$r_s$  is the radius from the central source of light to the surface of the sphere, and

$4\pi r_s^2$  = the total area of the sphere in  $\text{cm}^2$ .

Since  $n_0$  and  $4\pi$  are constants,  $I_0$  is seen to vary as  $1/r_s^2$ . This is the inverse square law.

For a radioisotope, whose half-life is extremely long compared to the time taken to implement the series of measurements in this experiment,  $n_0$  is synonymous with the activity,  $A_0$ , of the radioactive source. Consequently, Equation (18) can be expressed as

$$r_0 = \frac{N}{T} = A_0 \frac{a_d}{4\pi r_s^2} \epsilon_{\text{int}} \quad (19)$$

Where

$r_0$  is the true counting rate derived from the GM tube,

$N$  is the number of counts measured in the counting time  $T$  (corrected for dead time and background),

$A_0$  is the activity of the radioactive source,

$\epsilon_{\text{int}}$  is the intrinsic efficiency of the GM tube for detecting the gamma rays,

$a_d$  is the effective sensitive area of the detector at its entrance window, and

$r_s$  is the distance from the point source to the entrance window of the detector.

The factors in Equation (19) can be understood as follows. The effective sensitive area at the input to the detector intercepts a fraction,  $a_d/4\pi r_s^2$ , of the total area of the sphere of radius  $r_s$ . Consequently, the detector intercepts that same fraction of the isotropic radiation emitted by the source. Only a fraction,  $\epsilon_{\text{int}}$ , of the photons impinging on the sensitive area of the detector window are actually counted by the detector, due to window attenuation and the efficiency of converting photons into ionized atoms in the GM tube. For more details, see the section on Detection Efficiency in reference 1.

The purpose of this experiment is to verify the  $1/r_s^2$  dependence predicted by Equation (19).

#### Procedure

1. Set the GM tube at the proper operating voltage, and place the  $1\text{-}\mu\text{Ci } ^{60}\text{Co}$  source 1 cm away from the face of the window.
2. Count for a period of time long enough to get reasonable statistical precision ( $\geq 4000$  counts).
3. Move the source to 2 cm, and repeat the measurement for the same amount of time. Continue for the distances listed in Table 2.1. (Note that for the longer distances the time will have to be increased to obtain the minimum number of counts necessary for adequate statistical precision.)



### EXERCISES

- a. Correct the counts,  $N$ , first for dead time and then for background, and fill in the corrected counting rate in Table 2.1.
- b. On linear graph paper, plot the corrected counting rate ( $y$  axis) as a function of distance ( $x$  axis). This plot should have the  $1/r_s^2$  characteristics exhibited by Equations (18) and (19).
- c. Is there another way to plot this data so that the exponent for  $r_s$  in Equations (19) can be confirmed to be 2? See Experiment 2.4 for ideas. Try your idea, and report how closely the measured exponent approximates 2.

Table 2.1		
Distance	N/T (counts/min)	Corrected N/T (counts/min)
1 cm		
2 cm		
3 cm		
4 cm		
5 cm		
6 cm		
7 cm		
8 cm		

Equation (19) can be rearranged in the form

$$r_0 r_s^2 = \frac{A_0 a_d \epsilon_{int}}{4\pi} = K \tag{20}$$

Consequently, the product of the true counting rate and the square of the source-to-detector distance should be a constant for all positions.

### EXERCISES

- d. Multiply the corrected counting rates from Table 2.1 by the corresponding source-to-detector window distance to calculate  $K$  for each position. Plot the values for  $K$  ( $y$  axis) versus  $r_s$  ( $x$  axis) on linear graph paper. Choose a  $y$ -axis scale that permits observation of the scatter in the individual values of  $K$ .
- e. If Equations (18) through (20) are valid, the value of  $K$  should be constant. Is there a systematic change in the value of  $K$  versus the source-to-detector distance? Why? (See the section on Detection Efficiency in reference 1 for possible explanations.)
- f. Are there random fluctuations of  $K$  in the graph? What are the possible sources of those random variations?

## EXPERIMENT 2.6. Counting Statistics

### Purpose

As is well known, each measurement made for a radioactive sample is independent of all previous measurements, because radioactive decay is a random process. Repeated individual measurements of the activity vary randomly. However, for an ensemble comprising a large number of repeated, individual measurements, the deviation of the individual counts from what might be termed the "ensemble average count" behaves in a predictable manner. Small deviations from the average are much more likely than large deviations. In this experiment, we will see that the frequency of occurrence of a particular deviation from this average, within a given size interval, can be determined with a certain degree of confidence. Fifty independent measurements will be made, and some rather simple statistical treatments of the data will be performed. The experiment utilizes a  $^{60}\text{Co}$  source which has a half-life that is very long compared to the measurement time. The 5.26-year half-life ensures that the activity can be considered constant for the duration of the experiment.

## Experiment 2 Geiger Counting

### Relevant Equations

The average count for  $n$  independent measurements is given by

$$N_{av} = \frac{N_1 + N_2 + N_3 + \dots + N_n}{n} = \sum_{i=1}^n \frac{N_i}{n} \quad (21)$$

where  $N_1, N_2, N_3, \dots, N_n$  and  $N_i$  are the counts in the  $n$  independent measurements.

The deviation of an individual count from the mean is  $(N_i - N_{av})$ . Based on the definition of  $N_{av}$

$$\sum_{i=1}^n (N_i - N_{av}) = 0 \quad (22)$$

For cases where the percent dead time losses are small, it can be shown that the expected standard deviation,  $\sigma_N$ , can be estimated from

$$\sigma_N \approx \sqrt{N_{av}} \approx \sqrt{N_i} \quad (23)$$

with the estimate from  $N_{av}$  being more precise than the estimate from the individual measurement  $N_i$ . See references 10 and 11 for details. Thus,  $\sigma_N$  is the estimate of the standard deviation expected for the distribution of the measured counts,  $N_i$ , around the true mean.

Frequently, one is dealing with counting rates, rather than counts. If the true counting rate is defined by the number of counts accumulated in the counting time  $T$ , i.e.,

$$r_i = \frac{N_i}{T} \quad (24)$$

then, the estimated standard deviation in the counting rate can be calculated from

$$\sigma_r = \frac{\sigma_N}{T} \approx \frac{\sqrt{N_{av}}}{T} \approx \frac{\sqrt{N_i}}{T} = \sqrt{\frac{r_i}{T}} \quad (25)$$

A meaningful way to express the statistical precision of the measurement is via the percent standard deviation, which is defined by

$$\sigma\% = \frac{\sigma_r}{r_i} \times 100\% = \frac{\sigma_N}{N_i} \times 100\% = \frac{100\%}{\sqrt{N_i}} \quad (26)$$

Note that achieving a 1% standard deviation requires 10,000 counts.

### Procedure

1. Set the operating voltage of the Geiger tube at the value determined in Experiment 2.1.
2. Place the  $^{60}\text{Co}$  source far enough away from the window of the GM tube so that  $\sim 1000$  counts can be obtained in a time period of 0.5 min.
3. Without moving the source, take 50 independent 0.5 minute runs and record the counts for each run in Table 2.2. (Note that you will have to extend Table 2.2; only ten entries are illustrated.) The counter values,  $N_i$ , may be recorded directly in the table since, for this experiment,  $N_i$  is defined as the number of counts recorded for a 0.5 minute time interval.
4. With a calculator determine  $N_{av}$  from Equation (21). Fill in the values of  $N_i - N_{av}$  in Table 2.2. It should be noted that these values can be either positive or negative. You should indicate the sign in the data entered in the table.

### EXERCISES

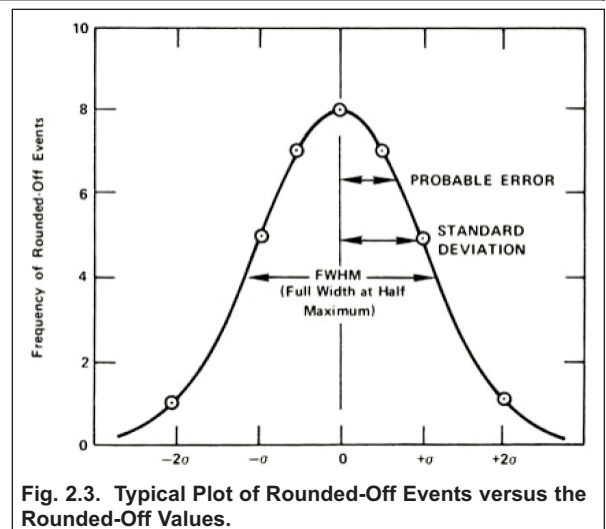
- a. Calculate  $\sigma_N$ , and fill in the values for  $\sigma_N$  and  $(N_i - N_{av})/\sigma_N$  in the table, using only two decimal places. Next, round off the values for  $(N_i - N_{av})/\sigma_N$  to the nearest 0.5 and record the values in the "Rounded Off" column of the table. Note that in Table 2.2 we have shown some typical values of  $(N_i - N_{av})/\sigma_N$  and the rounded-off values for guidance.
- b. Make a plot of the frequency of the rounded-off events  $(N_i - N_{av})/\sigma_N$  vs. the rounded-off values. Fig. 2.3 shows this plot for an ideal case. Note that at zero there are eight events, etc. This means that in our complete rounded-off data in Table 2.2 there were eight zeros. Likewise, there were seven values of +0.5, etc.
- c. Does your plot follow a normal distribution similar to that in Fig. 2.3?

Table 2.2*							
Run	$N_i$	$\sigma_N$	$N_i - N_{av}$	$(N_i - N_{av})/\sigma_N$		$(N_i - N_{av})/\sigma_N$ (Rounded Off)	
				Typical	Measured	Typical	Measured
1				-0.15		0	
2				+1.06		+1.0	
3				+0.07		0	
4				-1.61		-1.5	
5				-1.21		-1.0	
6				+1.70		+1.5	
7				-0.03		0	
8				-1.17		-1.0	
9				-1.67		-1.5	
10				+0.19		0	

\*Typical values of  $(N_i - N_{av})/\sigma_N$  and  $(N_i - N_{av})/\sigma_N$  Rounded Off; listed for illustrative purposes only.

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**Fig. 2.3. Typical Plot of Rounded-Off Events versus the Rounded-Off Values.**

Specifications subject to change  
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